

catool

Combustion Analysis TOOL

User Guide

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catool (www.catool.org)

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Introduction

Catool is an open-source software package designed to post-process cylinder pressure data collected from an internal combustion engine. As well as calculating standard thermodynamic calculations such as pressure rise rates, heat transfer and knock metrics, it deals with the format conversion, absolute pressure referencing, error detection and correction required for robust data analysis.

Catool can import raw combustion data from the industry standard AVL I-File format as well as ASCII text files, allowing the flexibility to use off-the-shelf data acquisition systems such as those from National Instruments and Measurement Computing, to standard combustion systems such as AVL Indiscope and ADAPT CAS.

Catool has been designed to be fast, flexible and capable of simple modification to suit a range of combustion analysis tasks. Its primary limitation is that of available memory, it does not have built-in limitations on the number of channels or lengths of data acquisition.

Downloading and Installation

The latest version of catool can be downloaded from www.catool.org. It is available both as source code for compilation on Unix type operating systems or pre-compiled binaries for Microsoft Windows.

By providing source code, catool can be compiled to run on almost any operating system and can easily be modified to suit particular purposes.

By providing pre-compiled binaries for Microsoft Windows, most users can be up and running in a matter of minutes.

Source code and Compiling

If you wish to install catool out-of-the-box on a Microsoft Windows platform you can skip to the Binary Installation section.

Compiling in a UNIX environment

Replace the # with the latest version number of catool:

```
mkdir /usr/src/catool-0.9.#
cd /usr/src/catool-0.9.#
wget http://catool.org/cgi-bin/download?catool-0.9.#-src.zip
unzip catool-0.9.#-src.zip
make clean
make
```

catool should now be built. Check by running

```
./catool
```

You should see the following output

```
Usage: catool [-h] [--debug-level=<debug_level>] file
```

Binary Installation

Once you have compiled or downloaded the catool binary this needs to be copied to a suitable working directory. On a UNIX platform this could be perhaps `/usr/bin`, on a Windows platform `C:\catool`. In a Windows environment you need to copy both the `catool.exe` and `pthreadGC2.dll` files to your computer.

Background

Crank Angle Domain

Unlike most traditional data analysis, combustion analysis is performed in the crank angle domain.

Data can be stored using up to five variable resolutions through an engine cycle. This minimises storage and processing, so for example, data can be stored at 1 degree resolution for the intake and compression strokes, 0.2 degree resolution for the first 40 degrees of the expansion stroke and the remaining at 1 degree resolution. A high resolution in this case enables robust knock detection.

Catool uses the following nomenclature regarding crank angles:

degrees – number of degrees from intake TDC, therefore:

Intake stroke - -360 to -180 degrees

Compression stroke – -180 to 0 degrees

Expansion stroke – 0 to 180 degrees

Exhaust stroke – 180 to 360 degrees

Note that in versions prior to 0.9.8 the nomenclature of degrees ranged from 0 to 720 for a four stroke cycle. This has been changed in line with general industry practice of -360 to +360 degrees. Existing configuration files will need their angular references modified by subtracting 360 degrees for a four stroke engine or 180 degrees for a two stroke engine.

theta – these are units of the lowest resolution supported by catool. As standard this is set as 1/20th of a degree. For a four stroke engine cycle theta therefore ranges from 0 to 14400 (720 degrees * 20 theta/degree).

crank_angle – data referenced in the crank_angle domain is in sequential order. This is important to understand when the crank angle abscissa has multiple measurement tables. For simple 1 degree resolution data crank_angle would range from 0 to 720. For data with a multiple resolution abscissa, for example 0 to 360 degrees @ 1 degree resolution, 360 to 400 degrees @ 0.2 degree resolution and 400 to 720 degrees @ 1 degree resolution there would be 880 crank_angle data points.

Cycles

Crank angle data is stored for a number of engine cycles. One cycle being two revolutions for a four stroke engine.

Catool's analysis is stored either in the crank angle domain, such as mean gas temperature and on a cyclic based, such as peak mean gas temperature. Catool also calculates statistics based over the entire set of engine cycles.

Configuration file

Catool is a command line driven program using a configuration file to configure the file input/output and analysis to be run. The configuration file is in text format and can be edited using Notepad or WordPad in Windows.

Usage

Catool is invoked from the command line.. This is either the Windows or UNIX command prompt .

Windows

Open the command prompt by opening Start, All Programs, Accessories, Command Prompt.

**Microsoft Windows XP [Version 5.1.2600]
(C) Copyright 1985-2001 Microsoft Corp.**

```
C:\>cd catool
```

```
C:\catool>catool rover_v8_csv.ccf
```

```
Combustion Analysis Tool (catool)  
Copyright (C) Ben Brown, 2000-2009  
Version 0.9.7  
www.catool.org
```

```
catool incorporates code from Pthreads-win32 (sourceware.org/pthreads-win32)
```

```
Copyright (C) 1998 John E. Bossom  
Copyright (C) 1999,2006 Pthreads-win32 contributors
```

```
Config File: rover_v8_csv.ccf  
Debug level: NOTICE  
Set input filetype to CSV  
Set input abscissa type to CRANK_ANGLE  
[cut]
```

```
Analysis complete
```

```
C:\catool>
```

Examples

The following examples use some combustion data recorded from a Rover V8 engine. The raw data is available at <http://sourceforge.net/projects/catool/files/sample-data/rover-v8/roverv8-2000rpm-wot.zip/download>.

Rover V8 CSV – Load, Analyse, Save

Download the example configuration from http://catool.org/files/rover_v8_csv.ccf.

Rover V8 AVL – Load, Analyse, Save

Download the example configuration from http://catool.org/files/rover_v8_avl.ccf. This uses the AVL I-File generated in the above CSV example.

Rover V8 AVL – Load, Save as MATLAB

Download the example configuration from http://catool.org/files/avl_matlab.ccf.

Loading IFile data directly into MATLAB

See here <http://catool.org/?q=node/2>.

Reference

The following are valid parameters in the catool configuration file:

debug-level (DEBUG | NOTICE | WARNING | ERROR | FATAL | SILENT)

This configures the amount of output catool displays on the screen ranging from very comprehensive (DEBUG) to nothing (SILENT). This parameter can also be specified at the command line:

```
catool -debug-level=WARNING config.ccf
```

In specifying the debug level as a command line option, any use in the config file will have no effect.

input-file-type (IFILE | CSV)

This specifies the type of file catool will be loading next.

input-file <filename>

This specifies the file which will be used to load data. You should specify the type of file with input-file-type before this command as catool will load the file header and initialise various data structures ready to load any channels.

input-abcissa-type (CRANK_ANGLE | CYCLE | TIME)

This specifies the abscissa type when loading CSV data.

input-data (CA | TIME | RTP_RESULTS | RESULTS_RAW | RESULTS | ALL)

number-of-channels <n>

This specifies the number of channels when loading CSV data.

number-of-cycles <n>

This specifies the number of cycles to load when loading CSV data.

If CSV or IFile data has already been loaded then this will modify the number of cycles as specified.

resolution <n>

This specifies the resolution of data when loading CSV data. It is either in samples per degree for crank angle data or Hertz for time based data depending on the setting of input-abcissa-type.

recording-length <n>

This specifies the recording length in seconds for CSV time based data as specified with input-abcissa-type.

header-lines <n>

This specifies the number of lines to ignore at the beginning of a file when loading CSV data.

load-channels (<channel-name> | none | all)

This loads the channels specified, either none, all or the channel name specified. The file must have been pre-loaded by specifying the input-file command.

channel (<channel-name> | all) name <name>
channel (<channel-name> | all) description <description>
channel (<channel-name> | all) units <units>
channel (<channel-name> | all) slope <slope>
channel (<channel-name> | all) offset <offset>
channel (<channel-name> | all) tdc_offset <tdc-offset>
channel (<channel-name> | all) cylinder <n>
channel (<channel-name> | all) type <type>

Valid channel types: ENG_SPEED, CYLPR, INJPR, NEEDLELIFT, VALVELIFT, INLETPR, EXHPR, LOWT, HIGHT, SPKPRICURR, SPKSECCURR, BLOCKACC, IGNANG, CPS, CID, INJECTOR, TIME, FRP, RESULTS, SCALAR, STATEVAR, CAN, DIGITAL, STD_RESULTS, LS_DAQ, TEMPERATURE, PRESSURE.

channel (<channel-name> | all) soc
start_window <angle>
finish_window <angle>
value <value>
channel <name>
aligned (0|1)
invert (0|1)
type (NONE | FIXED | CA_CHANNEL_RISE_RATE |
CA_CHANNEL_FALL_RATE | CA_CHANNEL_AVERAGE |
CYCLE_CHANNEL)

channel (<channel-name> | all) channel-offset
fixed_value <value>
start_window <angle>
finish_window <angle>
polytropic_index <index>
calc_interval <interval>
channel <name>
truncate <0|1>
type (NONE | FIXED | POLYTROPIC | WINDOW | WINDOW_ABS
| MEAN | OTHER)

channel (<channel-name> | all) abscissa
show
clear
resample
add <start-angle> <finish-angle> <resolution>

```
channel ( <channel-name> | all ) filter
  type <type>
  upper_frequency <hz>
  lower_frequency <hz>
```

engine

name <name>
evo_ca <n>
ivc_ca <n>
bore <n>
stroke <n>
conrod_length <n>
pin_offset (<cylinder-number> | all) <n>
compression_ratio <n>
cylinders <n>
strokes <2|4>
type (SI | DISI | CI_DI | CI_IDI | SI_ROTARY)
piston_factor <n>
info

load-file

set

comment
parameter-file
date
parameter (0-27) <name> <units> <value>
eeoc_start_window
eeoc_finish_window
fft_start_window
fft_finish_window
eeoc_index
t_ivc
temp_ref_ca
mfb_n
annand_a
t_wall
R
poly_exp_start_angle
poly_exp_finish_angle
poly_comp_start_angle
poly_comp_finish_angle
pkp_start_angle
pkp_finish_angle
pkp_smoothing_range
pkp_smoothing_resolution
cd_start_angle
cd_finish_angle
engine_speed
injector_start_window
injector_finish_window
max_number_of_injections
align_injections_to_tdc
misfire_imep
slowburn_imep
knock_pkp
tla_range
wiebe_a_start
wiebe_a_finish
wiebe_a_step
wiebe_m_start
wiebe_m_finish
wiebe_m_step
engine_speed_channel

knock_integral_type (RECTIFIED_INTEGRAL |
SQUARED_INTEGRAL)

engine_speed_type (NOTHING | SPECIFY | CHANNEL_AVERAGE
| IFILE)

heat_release_mode 1 (FIRST_LAW | POLY_FIRST)

heat_transfer_model (ANNAND | WOSCHNI | HOHENBERG |
EICHELBERG | NUSSELT | BRILING)

output-file <filename> (force-overwrite)

Specifies the filename of the output file. Use 'force-overwrite' to be able to set the input and output filename the same.

output-file-type (IFILE | CSV | MATLAB)

Specified the file type of the output file.

output-data (CA_RAW | CA | CA_TO_TIME | TIME | RTP_RESULTS | RESULTS | RESULTS_RAW | ALL)

Specifies which types of data to output.

Analyse

all

none

BURN_ANGLE_1,2,5,10,20,25,50,75,80,90,95,98,99

BURN_DURATION_0_2,_0_5,0_10,0_90,2_90,5_90,10_90

CENTRE_OF_GRAVITY

CID_ANGLE

DWELL_TIME

EEOC

END_OF_INJECTION_1,2,3,4,5,6

ENGINE_SPEED

EXHAUST_VALVE_CLOSING

EXHUAST_VALVE_OPENING

GROSS_IMEP

INDICATED_TORQUE

INJECTOR_DURATION

INLET_VALVE_CLOSING

INLET_VALVE_OPENING

KNOCK_BOSS_FACTOR

KNOCK_BOSS_INTEGRAL

KNOCK_FACTOR

KNOCK_INTEGRAL

KNOCK_C_AND_D

KNOCK_C_AND_D_CA

LOWER_PUMPING_IMEP

MAX_BURN_RATE

MAX_BURN_RATE_CA

MAX_COIL_CURRENT

MAX_HEAT_RELEASE_RATE

MAX_HEAT_RELEASE_RATE_CA
MAX_MEAN_GAS_TEMP
MAX_MEAN_GAS_TEMP_CA
MAX_PRESSURE
MAX_PRESSURE_CA
MAX_PRESSURE_RISE_RATE
MAX_PRESSURE_RISE_RATE_CA
MIN_PRESSURE
MIN_PRESSURE_CA
MISSING_TOOTH_1
MISSING_TOOTH_2
MISSING_TOOTH_RATIO_MAX
MISSING_TOOTH_RATIO_MIN
NET_IMEP
NUMBER_OF_INJECTIONS
PRESSURE_OFFSET
PEAK_KNOCKING_PRESSURE
PEAK_KNOCKING_PRESSURE_CA
POLY_COMP
POLY_EXP
PUMPING_IMEP
START_OF_COMBUSTION
START_OF_INJECTION_1,2,3,4,5,6
TLA
TOOTH_GAP_RATIO_MAX
TOOTH_GAP_RATIO_MIN
TOTAL_HEAT_RELEASE
UPPER_PUMPING_IMEP
WIEBE_A
WIEBE_M
FFT
ANGULAR_TORQUE
DIGITAL_SIGNAL
GAMMA
GROSS_HEAT_RELEASE_RATE
H_COEFF
KNOCKING_PRESSURE
MEAN_GAS_TEMP
MFB
MOTORED_PRESSURE
MOVING_PRESSURE_AVERAGE
NET_HEAT_RELEASE_RATE
POLYFIT

POLYTROPIC_INDICES
PRESSURE_RISE_RATE
TOOTH_SPEED
WIEBE_MFB

Multiple items can be specified on one line. Use of a minus sign before the analysis type negates that analysis.

Example:

```
analyse all -WIEBE_A -WIEBE_M
```

Would run configure catool to run all analysis except Wiebe a and m parameter calculations. Because the analysis does not run until the run-analysis command you can specify numerous lines to build up the necessary analysis configuration.

run-analysis

Runs the analysis configured using analyse.

output

This outputs to the file specified by output-file the data specified with output-data in the file type specified by output-file-type.

print <message>

This displays the text specified so long as the debugging level is not set to FATAL or SILENT.

stop

Immediately stops the parsing of the config file.

threads <n>

Specifies the number of threads that catool should use. This should be set to the total number of processor cores the machine has.

Calculations

Heat Release

Burn rate analysis is commonly used with spark ignition engines to determine the mass fraction burned. Rassweiler and Withrow¹³ developed a technique in 1938 that is still considered today to be both accurate and computationally efficient.

During combustion, the pressure rise, Δp , during a crank interval, $\Delta\theta$, is considered to consist of pressure rise due to combustion, Δp_c , and pressure change due to change in volume, Δp_v .

$$\Delta p = \Delta p_c + \Delta p_v$$

As the crank angle increments from θ_i to θ_{i+1} the volume changes from V_i to V_{i+1} and the pressure from p_i to p_{i+1} . Assuming that the change in pressure due to volume change can be calculated from a polytropic process of constant k :

$$p_{i+1} - p_i = \Delta p_c + p_i \left[\left(\frac{V_i}{V_{i+1}} \right)^k - 1 \right]$$

hence

$$\Delta p_c = p_{i+1} - p_i \left(\frac{V_i}{V_{i+1}} \right)^k$$

Because the combustion process does not occur at constant volume, the pressure rise rate due to combustion is not directly proportional to the mass of fuel burned. Therefore the pressure rise due to combustion must be referenced to a datum volume, such as that at TDC, V_{tdc} .

$$\Delta p_c^* = \Delta p_c \frac{V_i}{V_{tdc}}$$

By identifying the end of combustion and the number of crank angle intervals between start and finish of combustion, N , the mass fraction burned can be calculated:

$$mfb = \frac{\sum_0^i \Delta p_c^*}{\sum_0^N \Delta p_c^*}$$

Cycle Based Calculations:

B0002	Angle between SOC (Start of Combustion) and 2% MFB
B0010	Angle between SOC and 10% MFB
B0090	Angle between SOC and 90% MFB
B0590	Angle between 5% and 90% MFB
B1090	Angle between 10% and 90% MFB
MFB01	1% MFB
MFB02	2% MFB
MFB05	5% MFB
MFB10	10% MFB
MFB20	20% MFB
MFB25	25% MFB
MFB50	50% MFB
MFB75	75% MFB
MFB80	80% MFB
MFB95	95% MFB
MFB98	98% MFB
MFB99	99% MFB
MXBRN	Maximum Burn Rate (J/deg)
AMXBRN	Angle of Maximum Burn Rate (deg)

Crank Angle Calculations:

MFB Mass Fraction Burned (%)

CID

Crank angle of the camshaft ID position

EEOC

The estimated end of combustion (EEOC) is required for determining the normalising value for mass fraction burned and for heat release analysis. There have been several methods suggested by researchers, but the most common is to determine the crank angle that provides a maximum value of equation 4.11.

$$x = p.V^{1.15}$$

In order to reduce the effects of signal noise, the method is modified slightly to determine the crank angle that provides a maximum over a five-point summation of equation 4.11:

$$x = \sum_{i=\theta-2}^{i=\theta+2} p_i.V_i^{1.15}$$

In order to ensure the end of combustion is not underestimated, ten degrees is added to the crank angle at which x reaches a maximum.

Start and End of Injection

Calculated from injector signal.

Cycle based calculations:

EOI1_	1 st End of Injection (deg)
EOI2_	2 nd End of Injection (deg)
EOI3_	3 rd End of Injection (deg)
EOI4_	4 th End of Injection (deg)
EOI5_	5 th End of Injection (deg)
EOI6_	6 th End of Injection (deg)
SOI1_	1 st Start of Injection (deg)
SOI2_	2 nd Start of Injection (deg)
SOI3_	3 rd Start of Injection (deg)
SOI4_	4 th Start of Injection (deg)
SOI5_	5 th Start of Injection (deg)
SOI6_	6 th Start of Injection (deg)

N

Engine Speed calculated over one engine cycle. Either two revolutions for a four stroke engine or one revolution for a two strong engine.

Mean Gas Temperature

The mean gas temperature is required for the calculation of heat release.

for a polytropic process²⁸

$$pV^n = \text{constant}$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{V_1}{V_2}\right)^{n-1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

hence

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{n-1} = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

for a known reference location, such as inlet valve closure:

$$p_{ref} \cdot V_{ref} = n \cdot R \cdot T_{ref}$$

rearranging gives:

$$\frac{T_{ref}}{p_{ref} \cdot V_{ref}} = \frac{1}{n \cdot R}$$

to calculate the temperature at an arbitrary position between inlet valve closure and exhaust valve opening:

$$T_{calc} = p_{calc} \cdot V_{calc} \cdot \frac{1}{n \cdot R}$$

assuming n and R remain constant, equation 4.35 can be substituted into equation 4.36:

$$T_{calc} = p_{calc} \cdot V_{calc} \frac{T_{ref}}{p_{ref} \cdot V_{ref}}$$

Cycle calculations:

MXT Maximum Mean Gas Temperature (K)
 AMXT Angle of Maximum Mean Gas Temperature (deg)

Crank Angle calculations:

MGTEMP Mean Gas Temperature (K)

EVC, EVO, IVC, IVO

Exhaust Valve Closing, Exhaust Valve Opening, Intake Valve Closing, Intake Valve Opening

IMEP

The combustion analysis software calculates indicated mean effective pressure using the following equation:

$$imep = \frac{\Delta\theta}{V_s} \sum p \cdot \frac{dV}{d\theta}$$

This equation has been shown to be both computationally efficient and provide good robustness to coarse crank angle resolutions¹⁴.

Using the formula between -180 and +180 degrees provides the gross imep and, outside of this range, the pumping imep. The addition of these two parameters provides the net imep that includes the input of energy from combustion and the losses due to pumping.

Additionally, the pumping imep is split at atmospheric pressure between the upper and lower pumping loop imep. The upper pumping loop indicates the amount of energy required to propel the combustion products through the exhaust valve and piping system. The lower loop indicates the losses due to induction, including the throttling losses across the intake valve³³.

Cycle calculations:

GMEP	Gross IMEP (-180 to 180 degrees)
NMEP	GMEP - PMEP
UPMEP	Upper pumping IMEP (-360 to -180 and 180 to 360 degrees)
LPMEP	Lower pumping IMEP (-360 to -180 and 180 to 360 degrees)
PMEP	UPMEP + LPMEP

QRTMAX, AQRMTX

Heat release analysis is generally applied to compression ignition engines, although there is no reason why it cannot be used in spark ignition applications. Heat release analysis computes how much heat would need to have been added to the cylinder contents, in order to produce the observed pressure variations⁸.

Using the first law of thermodynamics it can be shown^{6,8,21}:

$$\frac{dQ_{net}}{d\theta} = \frac{\gamma}{\gamma-1} p \frac{dV}{d\theta} + \frac{1}{\gamma-1} V \frac{dp}{d\theta}$$

where

γ is the ratio of specific heats

Q_{net} is the net heat release rate in Joules per degree

P is the in-cylinder pressure in Pascals

V is the in-cylinder volume in cubic metres

By taking into account the effects of heat transfer to the cylinder walls, the gross heat release can be calculated:

$$\frac{dQ_{gross}}{d\theta} = \frac{dQ_{net}}{d\theta} + \frac{dQ_{ht}}{d\theta}$$

$$\frac{dQ_{ht}}{d\theta} = h(T - T_{wall}) \frac{dA}{d\theta}$$

where

h is the heat transfer coefficient

T is the mean gas temperature in Kelvin, calculated from the equation of state ($pV=mRT$)

T_{wall} is the mean cylinder wall temperature in Kelvin

A is the instantaneous heat transfer surface area of the combustion chamber in cubic metres

Heat Transfer Coefficients

Over the years various papers have been published aiming to quantify the heat transfer coefficient to easily measured or derived engine parameters. Some of the most common functions used are implemented in the combustion analysis software and are presented below.

Hohenberg²³

$$h = 129.8V^{-0.06} p^{0.8} T^{-0.4} (\bar{v}_p + 1.4)^{0.8}$$

Woschni²⁴

$$h = 129.8B^{-0.2} p^{0.8} T^{-0.53} \left(C_1 \bar{v}_p + C_2 \frac{V_s T_{ref}}{P_{ref} V_{ref}} (p - p_{motored}) \right)^{0.8}$$

where

$C_1 = 6.18$ in scavenging period

$C_1 = 2.28$ in compression, combustion and expansion
 $C_2 = 0$ in scavenging period and compression
 $C_2 = 3.24 \times 10^{-3}$ in combustion and expansion
 $C_2 = 6.22 \times 10^{-3}$ in combustion and expansion (IDI engines)⁸

Annand²⁵

$$h = \frac{a \cdot \lambda}{B} \text{Re}^{0.7} + c \frac{(T^4 - T_{wall}^4)}{T - T_{wall}}$$

where

$0.35 < a < 0.8$
 $c = 0$ during intake and compression
 $c = 0.576$ for CI engine combustion and expansion
 $c = 0.075$ for SI engine combustion and expansion
 from Bosch²⁷ $\circledast = 5.67 \times 10^{-8} \text{ W.m}^{-2}.\text{K}^{-4}$

from Street et al²⁶

$$\text{Re} = \frac{V \cdot d \cdot \rho}{\mu}$$

where

V is the mean velocity in the pipe (mean piston speed) in metres per second

d the characteristic length (engine bore) in metres

ρ is the density of the fluid in kilograms per cubic metres

μ is the dynamic fluid viscosity in kilograms per metre per second

Mean piston speed is calculated from engine speed. The software provides two sources of engine speed. The user can either specify it directly or if the engine speed channel has been recorded then the channel number can be specified and the speed averaged over the individual engine cycle periods. Recording engine speed has the benefit of accounting for the continually changing engine speed due to cycle-to-cycle variation of combustion.

$$V = 2 \cdot L \cdot N$$

where

N is engine speed in revolutions per second

L is engine stroke in metres

$$\rho = \frac{p}{R.T}$$

Annand²⁵ approximates:

$$\lambda = \frac{C_p \cdot \mu}{0.7}$$

from Brunt⁶:

$$C_p = \frac{R}{1 - \frac{1}{\gamma}}$$

$$\gamma = 1.338 - 6.0 \times 10^{-5} \cdot T + 1.0 \times 10^{-8} \cdot T^2 \quad (\text{based on gasoline engine})$$

The following figures are published:

Brunt:

$a=0.45$

$R=288.8$

Annand:

$\omega=4.702 \times 10^{-7} \cdot T^{0.645}$

$R=241.1$